2023 AB1/BC1

(a) $\int_{60}^{135} f(t)\,dt$ gives the amount of gasoline in gallons pumped into the tank between 60 and 135 seconds

$$\int_{60}^{135} f(t)\,dt \approx 30(0.15) + 30(0.1) + 15(0.05)$$
$$= 8.25$$

(b) f is continuous since it is differentiable. By the mean value theorem there would be a c value since $\frac{f(120) - f(60)}{120 - 60} = \frac{.1 - .1}{60} = 0$.

(c) $$g_{avg} = \frac{1}{150}\int_0^{150} g(t)\,dt$$
$$= .095 \text{ or } .096$$

(d) $g'(140) = -.004$ or $-.005$

The rate of flow of gasoline is decreasing at the rate of .005 gallons per second per second at time 140 seconds.

2023 AB 2

(a) $v(t) = 0$ stephen changes direction at $t=56$ seconds
$t = 56$ since $v(t)$ changes from positive to negative.

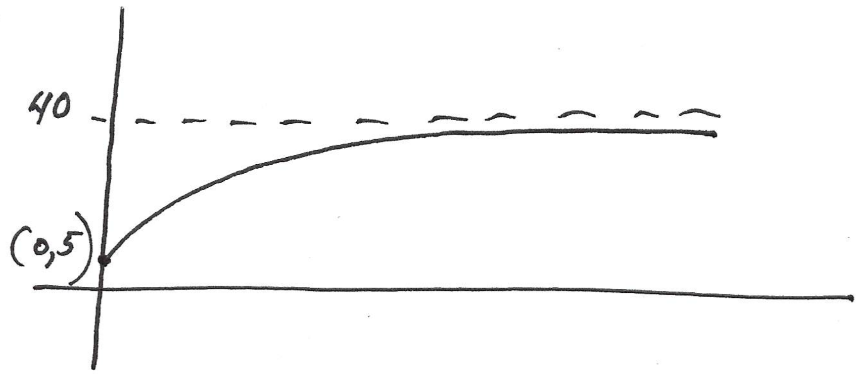
(b) $a(60) = v'(60)$ $v(60) < 0$
$= -.036 \ m/s^2$
Stephen is speeding up since $a(60) < 0$ and $v(60) < 0$.

(c) $\int_{20}^{80} v(t)\,dt = 23.383$ or 23.384

(d) $\int_{0}^{90} |v(t)|\,dt = 62.164$

2023 AB3 / BC3

(a)



(b) point (0,5) $\left.\frac{dM}{dt}\right|_{t=0} = \frac{1}{4}(40-5)$

$= \frac{35}{4}$

Tan. Line

$y - 5 = \frac{35}{4}(t-0)$

$M(2) \approx \frac{35}{4} \cdot 2 + 5$

$= 22.5$

(c) $\frac{d^2M}{dt^2} = -\frac{1}{4}\frac{dM}{dt}$

$= -\frac{1}{4} \cdot \frac{1}{4}(40-M)$

$\frac{d^2M}{dt^2} < 0$ on $[0,2]$ since $M < 40$

The approximation is an overestimate.

(d) $\frac{1}{40-M}\,dM = \frac{1}{4}\,dt$

$-\ln|40-M| = \frac{1}{4}t + C$

$-\ln 35 = C$

$-\ln|40-M| = \frac{1}{4}t - \ln 35$

$\ln|40-M| = -\frac{1}{4}t + \ln 35$

$40 - M = 35e^{-\frac{1}{4}t}$

$M = -35e^{-\frac{1}{4}t} + 40$

2023 AB 4 / BC 4

(a) f has neither a maximum nor a minimum since f' does not change signs.

(b) $(-2,0)$ and $(4,6)$ since f' is decreasing

(c) The limit is a $0/0$ indeterminate form, f is continuous since it is differentiable. $\lim_{x\to 2} f(x) = f(2)$

$$\lim_{x\to 2} \frac{6f(x)-3x}{x^2-5x+6} = \lim_{x\to 2} \frac{6f'(x)-3}{2x-5}$$

$$= \frac{-3}{-1}$$

$$= 3$$

(d) $f'(x) = 0$

$x = -1, 2, 6$

$f(x) = 1 + \int_2^t f'(t)\,dt$

x	$f(x)$
-2	$1+\frac{1}{2}\cdot 3\cdot 2-\frac{1}{2}\cdot 1\cdot 2 = 3$
-1	$1+\frac{1}{2}\cdot 3\cdot 2 = 4$
2	1
6	$1+\frac{1}{2}\cdot 2\cdot 2+\left(4-\frac{1}{4}\pi\cdot 2^2\right) = 7-\pi$
8	$1+\frac{1}{2}\cdot 2\cdot 2+\left(8-\frac{1}{2}\pi\cdot 2^2\right) = 11-2\pi$

The abs. min. value of f is 1.

2023 AB 5

(a) $h'(x) = f'(g(x))\,g'(x)$

$h'(7) = f'(g(7))\,g'(7)$

$= f'(0) \cdot 8$

$= \frac{3}{2} \cdot 8$

$= 12$

(b) $K''(x) = (f(x))^2 g'(x) + g(x) \cdot 2f(x) \cdot f'(x)$

$K''(4) = (f(4))^2 g'(4) + g(4) \cdot 2f(4) \cdot f'(4)$

$= 4^2 \cdot 2 + (-3) \cdot 2 \cdot 4 \cdot 3$

$= 32 - 72$

$= -40$

K is concave down at $x=4$ since $K''(4) < 0$

(c) $m(x) = 5x^3 + f(t)\Big|_0^x$

$= 5x^3 + f(x) - f(0)$

$= 5x^3 + f(x) - 10$

$m(2) = 5 \cdot 8 + f(2) - 10$

$= 40 + 7 - 10$

$= 37$

d. $m'(x) = 15x^2 + f'(x)$

$m'(2) = 15 \cdot 4 + f'(2)$

$= 60 - 8$

$= 52$

m is increasing since $m'(2) > 0$

2023 AB 6

(a) $6xy = 2 + y^3$

$6x\frac{dy}{dx} + y \cdot 6 = 3y^2\frac{dy}{dx}$

$\frac{dy}{dx}(6x - 3y^2) = -6y$

$\frac{dy}{dx} = \frac{-6y}{6x-3y^2}$

$= \frac{2y}{y^2-2x}$

(b) $\frac{dy}{dx} = 0 \rightarrow y = 0$

No horizontal tangent line exists since there is no point on the curve when $y = 0$.

(c) $\frac{dy}{dx}$ is undefined $\rightarrow y^2 = 2x$

$\frac{1}{2}y^2 = x$

$6\left(\frac{1}{2}y^2\right)y = 2 + y^3$

$3y^3 = 2 + y^3$

$2y^3 = 2$

$y^3 = 1$

$y = 1 \rightarrow x = \frac{1}{2}$ $\quad\left(\frac{1}{2}, 1\right)$

(d) $6x\frac{dy}{dt} + y \cdot 6\frac{dx}{dt} = 3y^2\frac{dy}{dt}$

$6 \cdot \frac{1}{2}\frac{dy}{dt} + (-2)6 \cdot \frac{2}{3} = 3(-2)^2\frac{dy}{dt}$

$3\frac{dy}{dt} - 8 = 12\frac{dy}{dt}$

$-8 = 9\frac{dy}{dt}$

$-\frac{8}{9} = \frac{dy}{dt}$ at $\left(\frac{1}{2}, -2\right)$

2023 BC 2

(a) $\frac{dy}{dt} = 2\cos t$

$a(t) = \langle x''(t), y''(t) \rangle$

$a(1) = \langle -1.444, -1.682 \rangle$ or $\langle -1.444, -1.683 \rangle$

(b) $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 1.5$

$t = 1.254$ (first time)

(c) $\frac{dy}{dx} = \frac{2\cos t}{e^{\cos t}}$

$\left.\frac{dy}{dx}\right|_{t=1} = \frac{2\cos 1}{e^{\cos 1}}$

$= .629$ or $.630$

$x(1) = 1 + \int_0^1 e^{\cos t}\,dt$

$= 3.341$ or 3.342

(d) $\int_0^{\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = 6.034$ or 6.035

2023 BC 5

(a) $g(x) = f(x)$

$x = 0, 3$

$$\text{area} = \int_0^3 f(x)\,dx - \int_0^3 g(x)\,dx$$

$$= 10 - \int_0^3 \frac{12}{3+x}\,dx$$

$$= 10 - 12\ln|3+x| \Big|_0^3$$

$$= 10 - 12\ln 6 + 12\ln 3$$

$$= 10 - 12\ln 2$$

(b) $\lim\limits_{b\to\infty} \int_0^b (g(x))^3\,dx$

$\lim\limits_{b\to\infty} \int_0^b 12^2 (3+x)^{-2}\,dx$

$\lim\limits_{b\to\infty} 12^2(-1)(3+x)^{-1}\Big|_0^b$

$\lim\limits_{b\to\infty} -12^2 \frac{1}{(3+b)} + \frac{12^2}{3}$

$0 + 48$

48

(c) $\int_0^3 x f'(x)\,dx$

$u = x$ $\quad dv = f'(x)\,dx$

$du = dx$ $\quad v = f(x)$

$x f(x)\Big|_0^3 - \int_0^3 f(x)\,dx$

$3 \cdot 2 - 0 - 10$

-4

2023 BC 6

(a) $f^{(4)}(x) = -2x\, f''(x^2)\cdot 2x + f'(x^2)(-2)$

$f^{(4)}(0) = -2f'(0)$ $\quad f''(0) = -f(0)$ $\quad f'''(0) = 0$

$= -6$ $\quad\quad = -2$

$f(x) \approx 2 + 3x - \frac{2}{2!}x^2 + 0 - \frac{6}{4!}x^4$

$= 2 + 3x - x^2 - \frac{1}{4}x^4$

(b) Lagrange Error $\le \frac{15(.1-0)^5}{5!} = \frac{1}{10^5 \cdot 8} < \frac{1}{10^5}$

(c) $g'(x) = e^x f(x)$ $\quad g'(0) = e^0 f(0) = 2$

$g''(x) = e^x f'(x) + f(x)e^x$ $\quad g''(0) = e^0 f'(0) + f(0)e^0 = 5$

$g(x) \approx 4 + 2x + \frac{5}{2}x^2$