Lesson 1.4 Squeeze Theorem, Limits of Compositions of Discontinuous Functions

Squeeze Theorem (Sandwich Theorem)

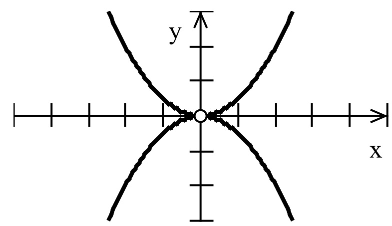
If $f(x) \le g(x) \le h(x)$ for all $x \ne c$ in some interval containing c and if $\lim_{x\to c} f(x) = L = \lim_{x\to c} h(x)$, then $\lim_{x\to c} g(x) = L$.

Informally: If a function g is squeezed (sandwiched) between two other functions with the same limit then g also approaches that same limit.

Examples:

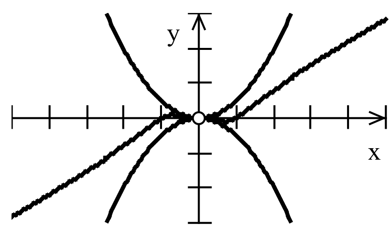
1. The graphs of $f(x) = \dfrac{x^3}{2x}$ and $g(x) = \dfrac{-x^3}{2x}$ are shown.

   Find $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$.



2. The graph of a third function $k(x)$ is shown along with the two functions from example 3.

   If $g(x) \le k(x) \le f(x)$ find $\lim_{x\to 0} k(x)$. Explain.



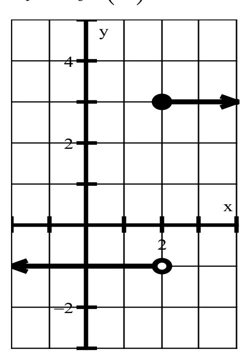
Use the functions graphed to find the following limits.

3. $\lim_{x\to 3} \dfrac{\left(f(x)\right)^2}{g(x)+1} =$

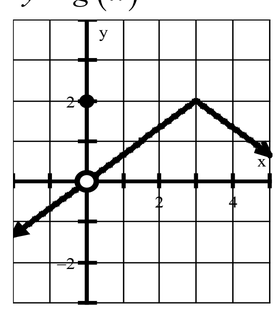
4. $\lim_{x\to 2.5} g\left(f(x)\right) =$

5. $\lim_{x\to 3} f\left(g(x)\right) =$

$y = f(x)$



$y = g(x)$



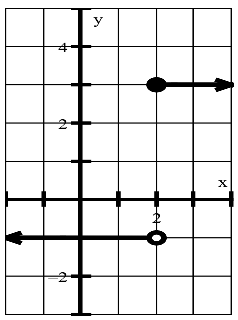
Assignment 1.4

1. If $f(x) = \dfrac{6x-18}{x-3}$ and $g(x) = \dfrac{6\sin\dfrac{\pi x}{6}}{\cos(x-3)}$ and it is known that $f(x) \le h(x) \le g(x)$ on the interval $[2,4]$ except at $x = 3$. Find $\lim_{x\to 3} h(x)$. Explain your reasoning.

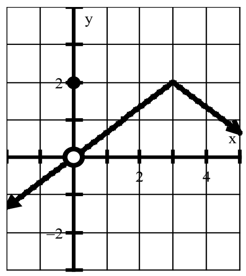
2. Given $f(x)=\dfrac{x^2-4}{x+2}$ and $f(x)\le h(x)\le j(x)$ for all x except $x=-2$. If $\lim\limits_{x\to -2} h(x)$ can be found by using the Squeeze Theorem what is $\lim\limits_{x\to -2} j(x)$?

Use the four functions graphed below to find the limits shown or state that the limit does not exist.

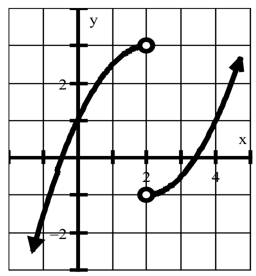
$y=f(x)$



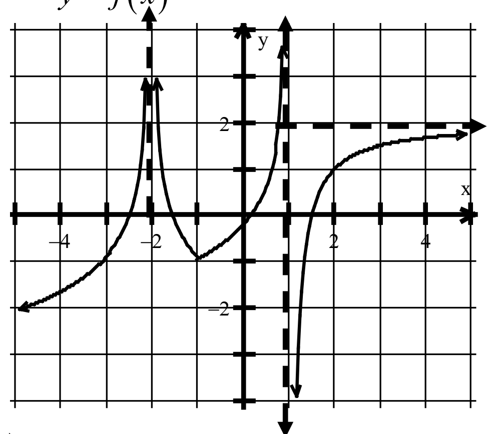
$y=g(x)$



$y=h(x)$



$y=j(x)$



3. $\lim\limits_{x\to -2} j(x)$

4. $\lim\limits_{x\to 1} j(x)$

5. $\lim\limits_{x\to -1} \dfrac{f(x)-2}{\left(j(x)\right)^2}$

6. $\lim\limits_{x\to \infty} h\left(j(x)\right)$

7. $\lim\limits_{x\to -1} g\left(f(x)+1\right)$

8. $\lim\limits_{x\to 0} f\left(|x|+2\right)$

9. $\lim\limits_{x\to 0}\left(g(x)\bullet f(x+2)\right)$

10. $\lim\limits_{x\to -2} j\left(j(x)\right)$

Selected Answers:
1. 6 2. -4 4. DNE 5. -3 7. 2 9. 0