Lesson 9.1 Series Definitions, Geometric Series, n^{th} Term Test

Factorial Definition: $n! = n(n-1)(n-2)(n-3)\bullet\bullet\bullet 1$

Example 1: Simplify $\dfrac{(n+2)!}{n!} =$

Definition: A **series** is a **sum** of numbers.

An infinite series can be represented as $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots + a_n + \cdots$

Example 2: Write out the first five terms of the series $\sum_{n=1}^{\infty} \dfrac{n}{n+1} =$

Examples: Write an expression for the nth term of the following series.

3. $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots$

4. $\dfrac{3}{1} + \dfrac{3}{2} + \dfrac{3}{6} + \dfrac{3}{24} + \dfrac{3}{120} + \cdots$

Examples: Write an expression for the following series using sigma notation.

5. $\dfrac{2}{1} + \dfrac{4}{3} + \dfrac{6}{5} + \dfrac{8}{7} + \cdots =$

6. $2 - 6 + 18 - 54 + 162 + \cdots =$

Example 7: What happens if we add more and more terms of a series like

$\sum_{n=0}^{\infty} (2n+1) = 1 + 3 + 5 + 7 + \cdots$ Show a sequence of partial sums.

This sequence of partial sums is approaching infinity. When this happens, the series is called **divergent**.

Example 8: What happens if we add more and more terms of $\sum_{n=1}^{\infty} \dfrac{3}{10^n} =$

This is an example of a **convergent** geometric series.

Geometric Series:
If consecutive terms in a series have a common ratio $\boldsymbol{r}$, the series is called a **geometric series**.

$\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + \cdots + ar^n + \cdots$ is the general form of a geometric series.

If the geometric series converges it is possible to find the sum even though it has infinitely many terms.

Let $S = a + ar + ar^2 + ar^3 + \cdots$

then $rS = \quad ar +$

subtracting $S - rS =$

factoring $S(1-r) =$

$S =$

$$\sum_{n=0}^{\infty} a r^n = \frac{a}{1-r}$$ **first term!** if the geometric series converges.

The Geometric Series Test: for a geometric series $\sum_{n=0}^{\infty} a r^n$

If $|r| \geq 1$ the geometric series **diverges**. If $|r| < 1$ the series **converges**.

Examples: Determine if these series converge or diverge and, if possible, find the sum of the series.

9. $\sum_{n=0}^{\infty} \frac{3}{2^n}$

10. $\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$

11. $\sum_{n=1}^{\infty} 4\left(-\frac{1}{2}\right)^n$

Example 12: Find the fraction form of the repeating decimal $.\overline{08}$ using a geometric series.

Example 13: The series $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right) = (1+1)+\left(1+\frac{1}{2}\right)+\left(1+\frac{1}{3}\right)+\cdots$ does **not** converge.
Show a sequence of partial sums.

A series **cannot** converge unless the terms approach a limit of **zero**.

n^{th} Term Test for Divergence:

If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges. This test is inconclusive if $\lim_{n\to\infty} a_n = 0$.

Example 14: Show that $\sum_{n=1}^{\infty} \frac{n!}{2n!+1}$ diverges.

Assignment 9.1

Simplify without using a calculator.

1. $\frac{7!}{10!}$

2. $\frac{(2n+1)!}{(2n-1)!}$

Write an expression for the *nth* term of each series. Use $n = 1,2,3,\cdots$

3. $\frac{3}{2}+\frac{4}{3}+\frac{5}{4}+\frac{6}{5}+\cdots$

4. $-1+\frac{1}{4}-\frac{1}{9}+\frac{1}{16}-\cdots$

5. $\frac{2}{1}+\frac{4}{3}+\frac{8}{7}+\frac{16}{15}+\cdots$

6. $\frac{3}{1}+\frac{3}{2}+\frac{3}{6}+\frac{3}{24}+\cdots$

Use sigma notation to write an equivalent expression for each series. Use $n = 0,1,2,3,\cdots$

7. $\frac{1}{2}+\frac{x}{6}+\frac{x^2}{24}+\frac{x^3}{120}+\cdots$

8. $-1+1+3+5+\cdots$

9. $\frac{1}{1}+\frac{4}{3}+\frac{9}{9}+\frac{16}{27}+\cdots$

Determine whether each of the following infinite series converges or diverges. Show justification and name the test being used. In addition, find the sum of the series, if possible.

10. $\sum_{n=0}^{\infty} 5\left(\frac{2}{3}\right)^n$

11. $\sum_{n=1}^{\infty} 5\left(\frac{2}{3}\right)^n$

12. $\sum_{n=0}^{\infty} 5\left(\frac{3}{2}\right)^n$

13. $\sum_{n=1}^{\infty} \frac{n^2+2}{n^2}$

14. $\sum_{n=2}^{\infty} \frac{n^2}{\ln n}$

15. $5+\frac{5}{2}+\frac{5}{4}+\frac{5}{8}+\cdots$

16. $3+\frac{9}{2}+\frac{27}{4}+\frac{81}{8}+\cdots$

17. $1+0.2+0.04+0.008+\cdots$

18. $\sum_{n=1}^{\infty} \frac{n}{\sin n}$

19. $\sum_{n=1}^{\infty} \frac{3^n+2}{3^{n+2}}$

20. $\sum_{n=0}^{\infty} \frac{e^n}{\pi^{n+1}}$

21. $\frac{1}{2}+\frac{2}{4}+\frac{6}{8}+\frac{24}{16}+\frac{120}{32}+\cdots$

22. $\sum_{n=1}^{\infty} \left(\sin e^{10}\right)^n$

23. $\sum_{n=1}^{\infty} \frac{(-5)^n}{6}$

24. $\sum_{n=1}^{\infty} \left(-\frac{5}{6}\right)^n$

25. $18-12+8-\frac{16}{3}+\frac{32}{9}-\cdots$

26. $\sum_{n=1}^{\infty} \frac{2n+3}{3n+2}$

27. $\sum_{n=0}^{\infty} \frac{n!}{e^n}$

28. $\sum_{n=0}^{\infty} \frac{4}{3^n}$

29. $\sum_{n=1}^{\infty} 4^{-n}$

30. Given the series $\sum_{n=1}^{\infty} \frac{n^2+2}{n^3}$:

   a. Find $\lim_{n\to\infty} a_n$.

   b. Explain why the *nth* Term Test **cannot** be used to conclude the series converges.

31. Find the x- and y-intercepts, relative extrema, and points of inflection for $y=\sin x+\cos x$ on $[0,2\pi)$. Then sketch the graph of y without using a calculator.

Differentiate implicitly to find $\frac{dy}{dx}$.

32. $\cos(y-x)=x^3+2$

33. $2xy=\tan y^2$

Find the limits without using a calculator.

34. $\lim_{x\to\frac{\pi}{2}} \left(\frac{\cos x}{x-\frac{\pi}{2}}\right)$

35. $\lim_{x\to 0} \frac{\sin(2x)}{\sin(-5x)}$

36. $\lim_{x\to 0} \frac{\sin(3x)}{x}$

37. $\lim_{t\to 0} (2t\sec t)$

38. Without using a calculator find $\int_0^{\infty} \frac{e^x}{1+e^x}\,dx$.

For Problems 39-42, a region is in the 1st quadrant bounded by $y = 3\cos(2x)$, $y = 3x$, and $x = 0$.

39. Use a calculator to find the area of the region. Show an integral set up and an answer

40. <u>Set up</u> (but do not integrate) an integral for finding the volume of the solid formed by revolving the region about the x-axis.

41. <u>Set up</u> (but do not integrate) an integral for finding the volume of the solid formed by revolving the region about $y = 4$.

42. <u>Set up</u> (but do not integrate) an integral for finding the volume of the solid formed by using rectangular cross sections whose bases are in the region and are perpendicular to the x-axis. The heights of the rectangles are always half of their bases.

Selected Answers:

1. $\frac{1}{720}$ 4. $\frac{(-1)^n}{n^2}$ 6. $\frac{3}{n!}$ 7. $\sum_{n=0}^{\infty} \frac{x^n}{(n+2)!}$ 10. converges by GST, Sum = 15

11. conv. by GST, Sum = 10 12. div. by GST or nTT 14. div. by nTT

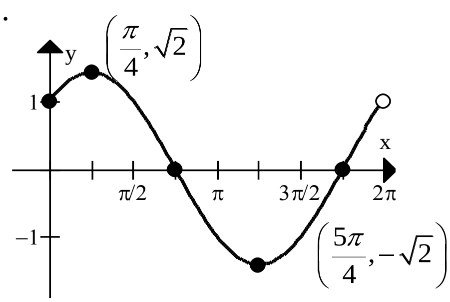
15. conv. by GST, Sum = 10 16. div. by GST or nTT 17. conv. by GST, Sum = 1.25

18. div. by nTT 20. conv. by GST, Sum = $\frac{1}{\pi - e}$ 22. conv. by GST, Sum = -.407 or -.408

23. div. by GST or nTT 24. conv. by GST, Sum = $-\frac{5}{11}$ 26. div. by nTT

27. div. by nTT 29. conv. by GST, Sum = $\frac{1}{3}$

31.



32. $y' = \frac{3x^2}{-\sin(y-x)} + 1 = \frac{3x^2 - \sin(y-x)}{-\sin(y-x)}$

33. $y' = \frac{-2y}{2x - 2y\sec^2 y^2}$ 34. -1 36. 3

38. diverges 39. .888 41. $\pi\int_0^{.5149} \left((4-3x)^2 - (4-3\cos(2x))^2\right)dx$

42. $\int_0^{.5149} \frac{1}{2}\left(3\cos(2x) - 3x\right)^2 dx$