LESSON 2.4 POSITION, VELOCITY, ACCELERATION

Important Terms	
Position Function	gives the location of an object at time t, usually $s(t)$, $x(t)$, or $y(t)$
Velocity Function	the rate of change (derivative) of position, usually $v(t)$ Velocity is positive for upward or rightward motion and negative for downward or leftward motion.
Acceleration Function	the rate of change (derivative) of velocity, usually $a(t)$
Initial Position	starting position (at $t=0$), s_o
Initial Velocity	starting velocity (at $t=0$), v_o
Speed	the absolute value of velocity
Displacement	the net change in position, (final pos. – original pos.)
Total Distance	total distance traveled by the object in the time interval (takes into account all direction changes)

Example 1. If $s(t)=t^3+t$, find $v(t)$ and $a(t)$.

Examples: Use the position function $s(t)=16t^3-36t^2+24$ of an object moving on a horizontal line for Examples 2-11. Distance units are measured in feet and time units are measured in seconds.

2. What is the initial position of the object?

3. What is the velocity of the object at $t = 1$ second?

4. What is the speed of the object at $t = 1$ second?

5. What is the acceleration of the object at $t = 1$ second?

6. When is the object at rest?

7. When is the object moving right?

8. When is the object moving left?

9. When is the velocity of the object

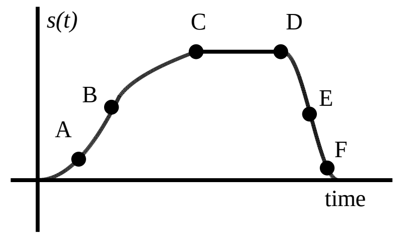
10. What is the displacement of the object

equal to 54 $^{ft}/_{sec}$?

between $t = 0$ and $t = 2$ seconds?

11. What is the total distance traveled by the object between $t = 0$ and $t = 2$ seconds?

The graph models the position function of a radio controlled model car. Answer these questions and explain.



12. Was the car going faster at A or at B?

13. When was the car stopped?

14. At which point was the car's velocity the greatest?

15. At which point was the car's speed the greatest?

Vertical Motion Examples:

Suppose $s(t) = -16t^2 + 48t + 160$ gives the position (in feet) above the ground for a ball thrown into the air from the top of a high cliff (where time is measured in seconds).

16. Find the initial velocity.

17. At what time does the ball hit the ground?

18. At what time does the ball reach its maximum height?

ASSIGNMENT 2.4

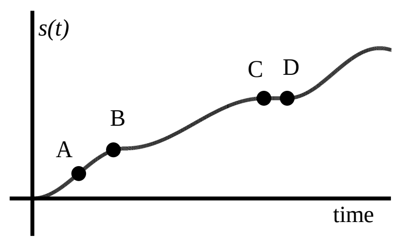
You may use a calculator for these questions.

1. The position, in meters, of a particle moving in a straight line is given by $x(t) = 4t^3 + 6t + 2.5$ (where t is measured in seconds).
   a. Find the velocity function.
   b. Find the velocity at time $t = 2$ seconds.
   c. Find the acceleration function.
   d. Find the acceleration at time 3 seconds.
   e. When is the velocity of the particle 18 meters per second?
   f. Find the velocity when the position of the particle is 25 meters.
   g. Find the initial position.
   h. Find the particle's displacement from 0 to 1.5 sec.

2. A helium balloon rises so that its height (position) is given by $s(t) = t^2 + 3t + 5$ (where height is measured in feet and time is measured in seconds). Assume $t \geq 0$.
   a. When is the balloon 45 feet high?
   b. How fast is the balloon rising at time 1 second?
   c. How fast is the balloon rising at time 4 seconds?
   d. What is the balloon's velocity when it is 45 feet high?

3. A ball rolls on an inclined plane with position function $s(t) = 2t^3 + 3t^2 + 5$ (where position is measured in centimeters and time is measured in seconds).
   a. Find the ball's velocity at time 2 seconds.
   b. When is the velocity of the ball 30 centimeters per second?

4. The graph at the right models the position function of a car. Answer these questions and explain each answer.
   a. What was the car's initial position?
   b. Was the car going faster at A or at B?
   c. Was the car speeding up or slowing down at B?
   d. What happened between C and D?

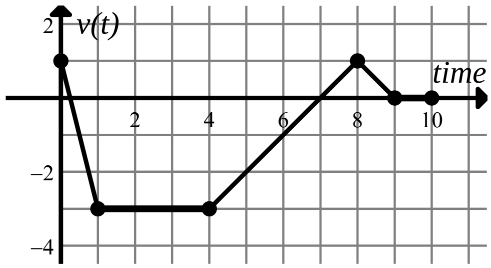


5. A particle moves along a horizontal line with position function $x(t) = t^3 - 3t^2$ (where position is measured in centimeters and time is measured in minutes).
   a. Find the particle's displacement between $t = 0$ minutes and $t = 5$ minutes.
   b. Find the particle's velocity when $t = 4$ minutes.
   c. Find the particle's acceleration when $t = 4$ minutes.
   d. At what time does the particle change direction?
   e. What is the total distance traveled by the particle between 0 and 5 minutes?

$$\textbf{Average Velocity} = \frac{\text{displacement}}{\text{elapsed time}} \qquad \textbf{Average Speed} = \frac{\text{total distance}}{\text{elapsed time}}$$

   f. Find the particle's average velocity (average rate of change of position) between $t = 0$ and $t = 5$ minutes.
   g. Find the particle's average speed between $t = 0$ and $t = 5$ minutes.

6. The graph at the right shows the velocity function of a particle moving horizontally.
   a. When does the particle move left?
   b. When is the particle's acceleration positive?
   c. When is the speed greatest?
   d. When does the particle stop for more than an instant?



7. The position at time t seconds of a pebble dropped from an initial height of 600 feet is given by $s(t) = -16t^2 + 600$.
   a. At what time will the pebble hit the ground?
   b. What is the pebble's velocity when it hits the ground?
   c. What is the pebble's speed when it hits the ground?

Find $f'(x)$ without using a calculator.

8. $f(x) = 2x - \dfrac{3}{x^3}$ 9. $f(x) = (2x+3)^2$ 10. $h(t) = 2\sin t - 3\ln t$ 11. $y = 2e^x + \ln x$

Evaluate the derivative of $f(x)$ at the indicated point without using a calculator.

12. $f(x) = 2x\sqrt{x}$ at $(4,16)$ 13. $f(x) = \sqrt[3]{x^2}$ at $(-8,4)$ 14. $f(x) = e\cos x - 9e^x$ at $x = 0$

15. If $y = x(x-2)$ find $\dfrac{d^2y}{dx^2}$.

16. Find an equation of a line tangent to the graph of $f(x) = 2x^4 - 3x^3$ when $x = 1$ without using a calculator.

17. Find a point on the graph of $f(x) = x^4 + 3$ where a tangent line has a slope of -4 without using a calculator

18. Find the x-value(s) where the function $f(x) = \begin{cases} \dfrac{x}{x^2-1}, & x \le 2 \\ x-1, & x > 2 \end{cases}$ is not differentiable. Give a reason for each x-value.

19. Use the limit definition of the derivative to find $f'(x)$ if $f(x) = 3x^2 - x$.

20. If $f(x) = x^3 + 5$, find the instantaneous rate of change at $x = 1$.

21. If $f(x) = x^3 + 5$, find the average rate of change between $x = 0$ and $x = 2$.